Hw 3 _ 2050b _ Sept 2022

1.* Solve the inequality system (in the sense to identify the "solution set" A, the set consisting of all x satisfying the inequality)

$$4 < |x+2| + |x-1| \leq 5$$

$\left(A = [-3, -\tfrac{5}{2}) \cup (\tfrac{3}{2}, 2]\right)$.

Hint: subdivide into subintervals in order to remove the absolute value signs.

2.* Show, by MI, the binomial formula:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

Hint: use $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$.

Show further that, $\forall a > 0$

$$(1+a)^n \geq 1 + na \quad \&$$

$$(1+a)^n \geq \frac{n(n-1)(n-2)}{3!} a^3 \quad \forall n \geq 3, \ \ldots$$

so $\frac{n^2}{(1+a)^n} \to 0$ & similarly $\frac{n^{10000}}{(1.000001)^n} \to 0$

3.* Let $x_1 > 0$ and $x_{n+1} := x_n + \frac{1}{x_n}$ $\forall n \geq 1$. Show that

(i) $x_{n+1} \geq x_n$ & $x_{n+1}^2 \geq x_{n+1} \cdot x_n = x_n^2 + 1$.

(ii) (x_n^2), and (x_n) are unbounded.

(iii) Can (x_n) converge?

Hint: either use the nece. condition theorem or by contradiction argument.

4.* Let

$$x_{n+1} = 2 + \frac{x_n}{2} \quad \forall n \geq 1$$

Then each of the following cases, is (x_n) monotone/convergent?

(i) $x_1 = 1$;

(ii) $x_1 = 10$;

Determine the limit when exists.